
# Heterogenous investors in energy system models

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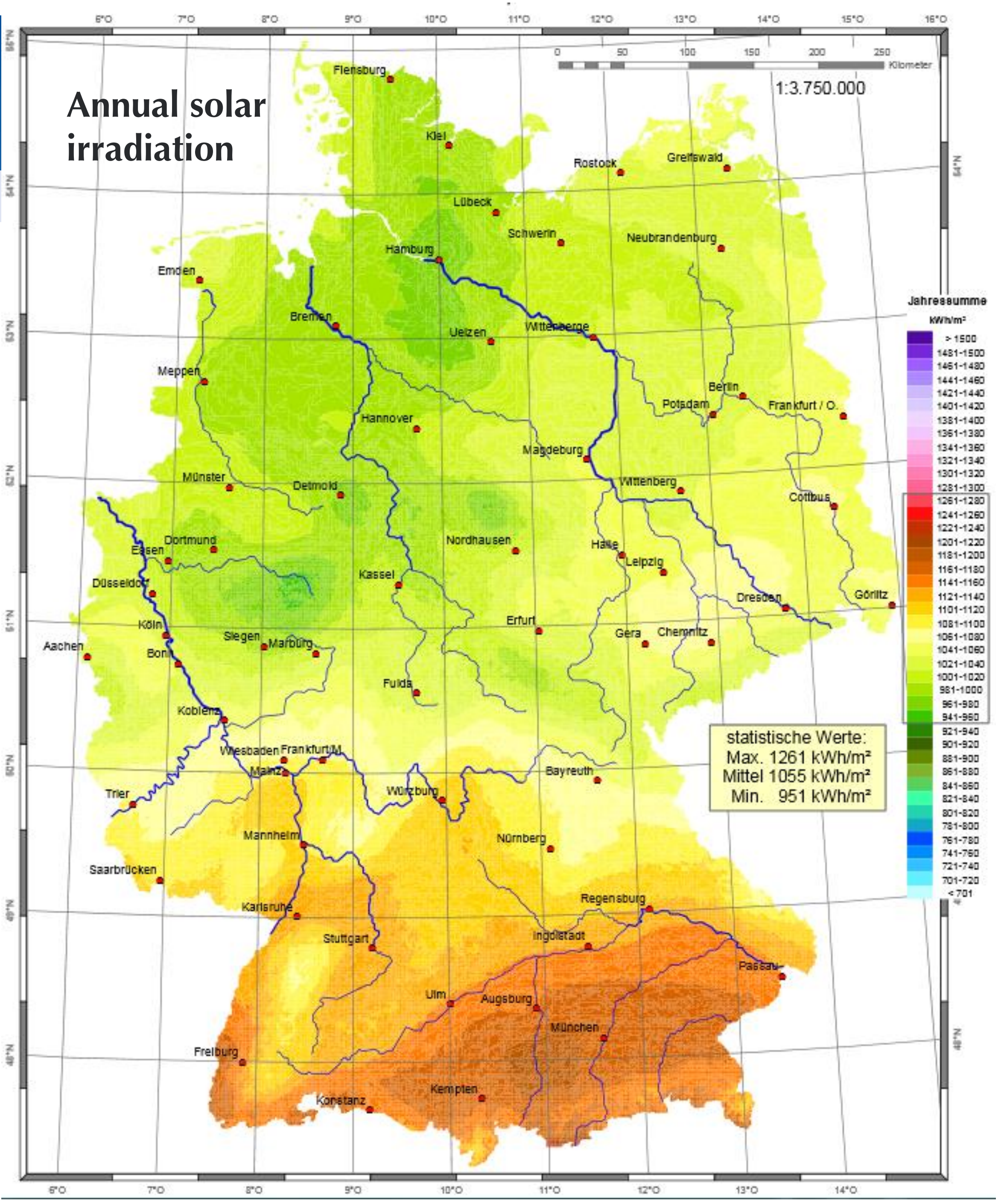
# Solar energy in Germany – the physical facts and the political questions

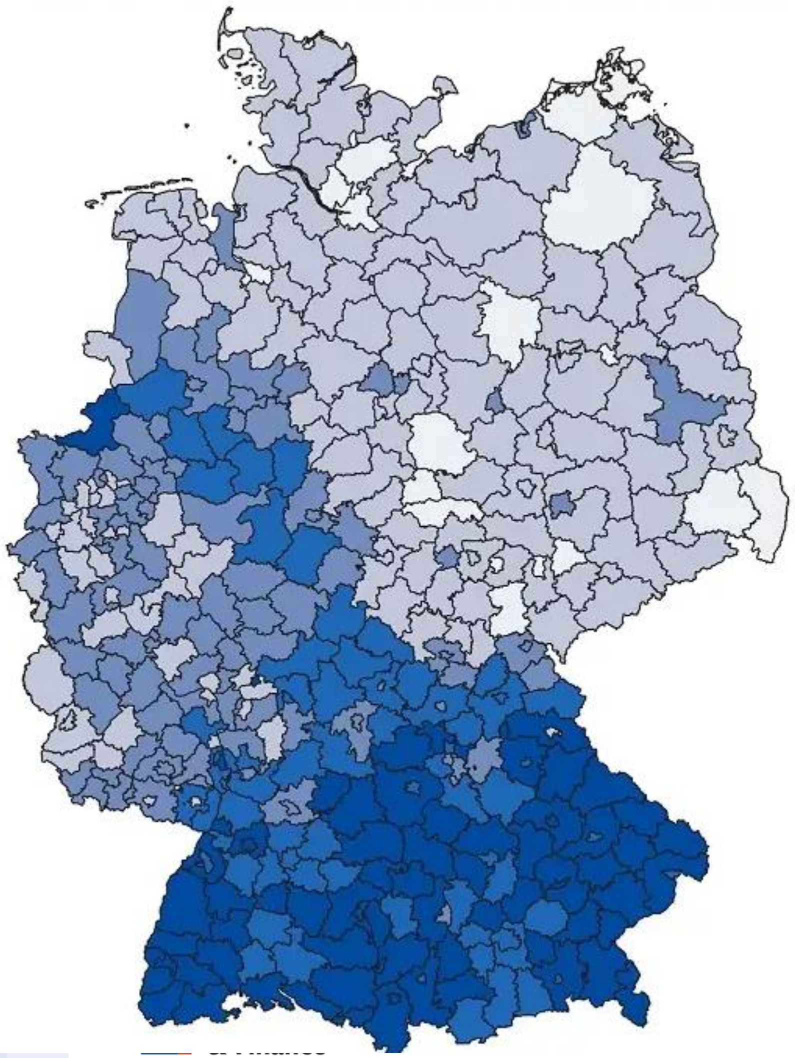
Motivation

- Where should investment in PV be made?
- Where are investments actually made?
- Who decides about investments?



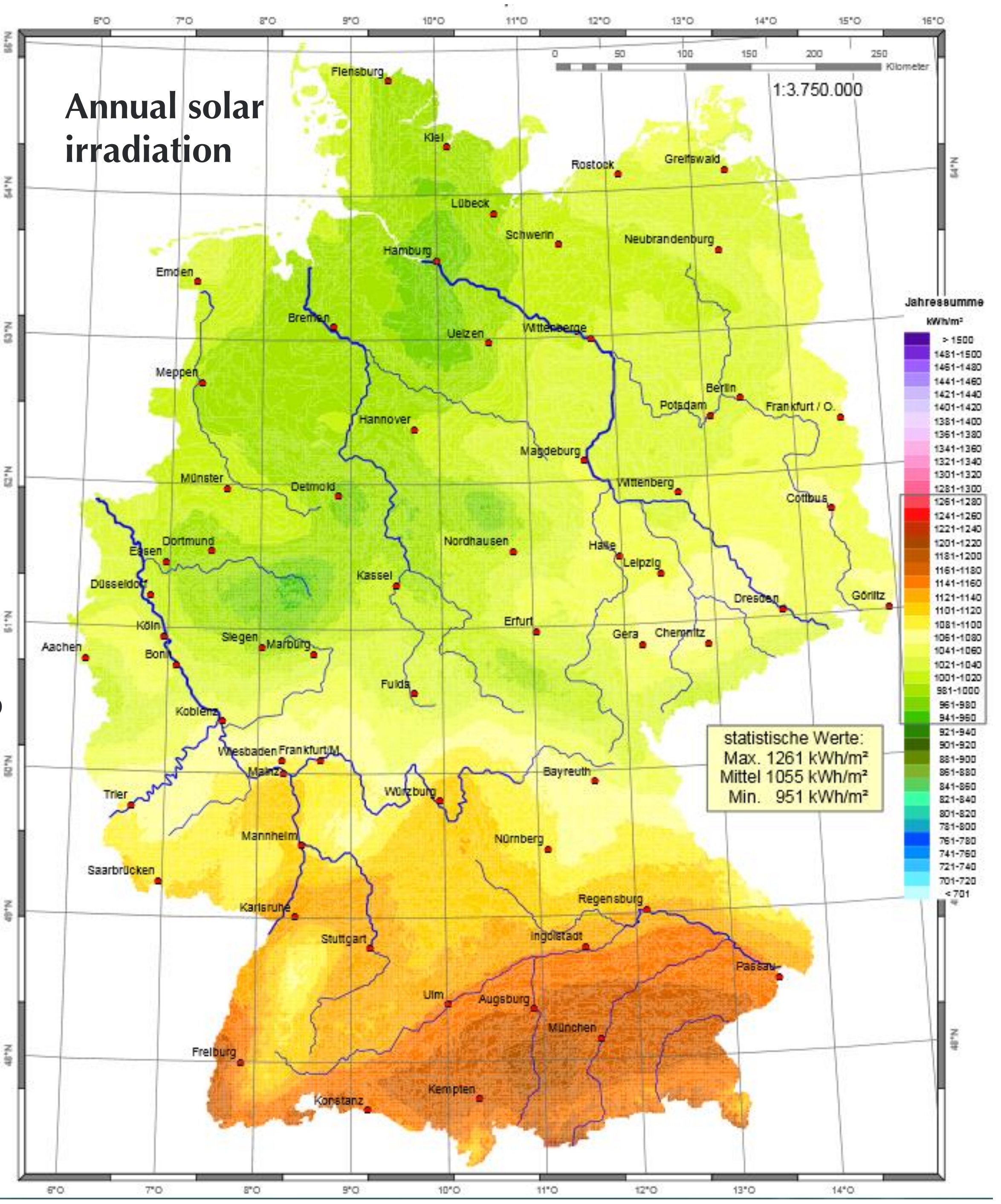
# Solar energy in Germany
# – the physical facts and the political questions





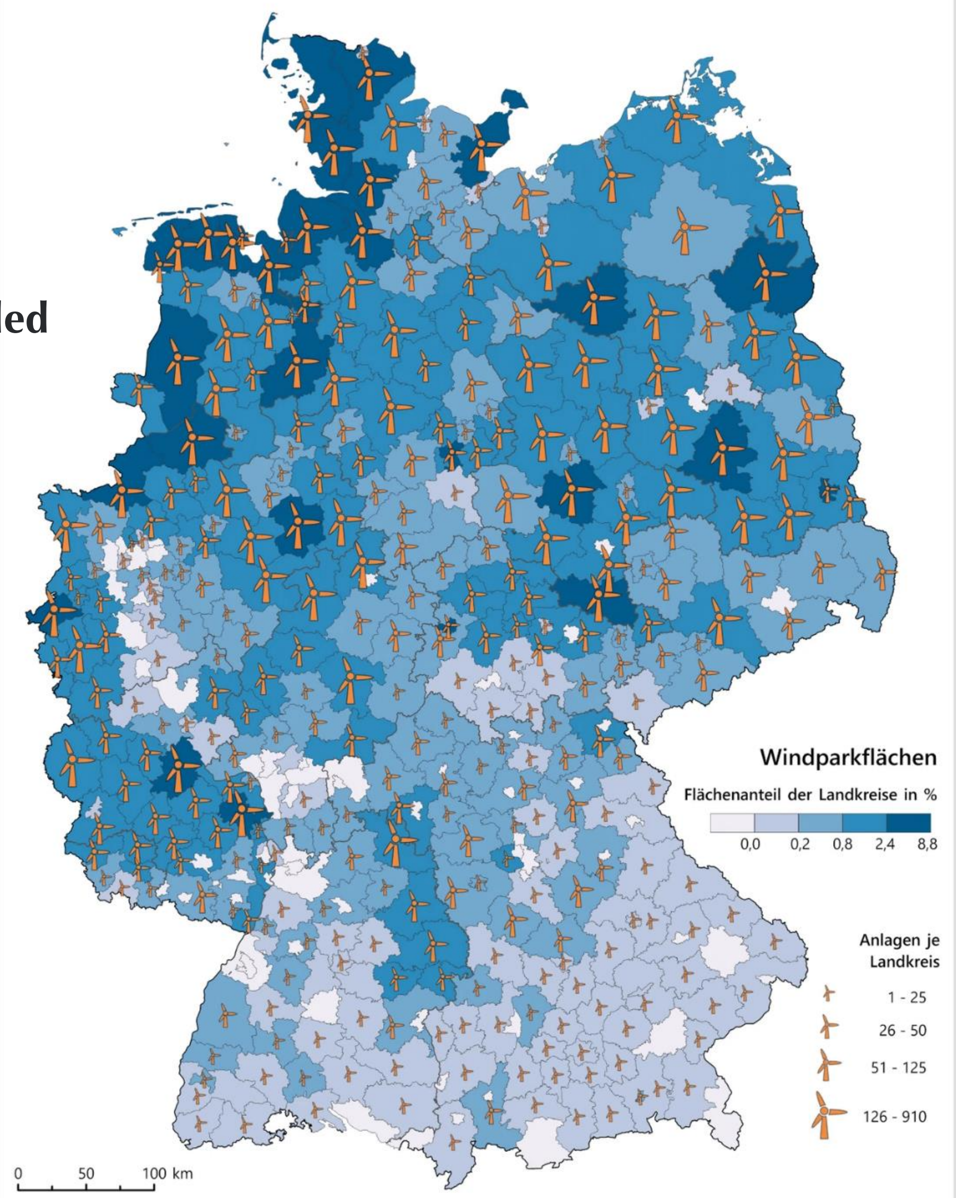
**Installed rooftop solar capacity 2018 – saturation with respect to available rooftoop areas**

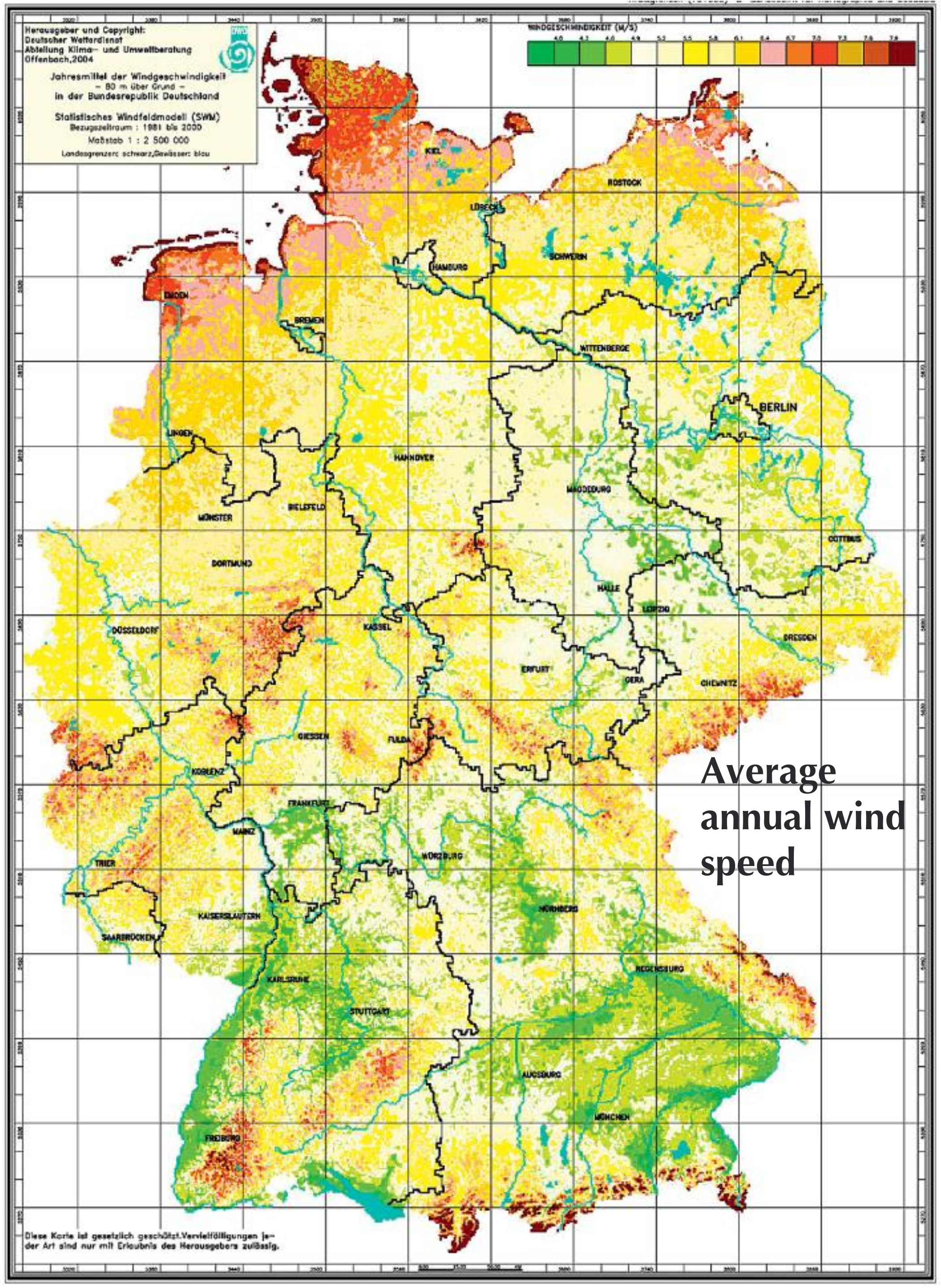




# Wind energy in Germany
# - same story but different

Number of installed wind turbines




Average annual wind speed

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# Motivation & objective

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## Motivation

- **Increasing shares of renewables**
  - Higher variations of net load within days (and between days)
  - ***More distributed generation***
    - at least for rooftop PV and in countries like Germany
  - Also **distributed flexibilities**
    - Notably electric vehicles
- **Heterogenous investments and investors**
  - Partly heterogeneity of technology potentials and preferences
  - Partly limited knowledge of planners/modellers
- **Standard energy system models do not cope with these investors**
  - Linear programs subject to penny switching
  - Differentiation by investment opportunities (sites and technology types) possible,
  - yet leads to large models and still unsatisfactory representation of individual decision making
- **Objective: develop an alternative approach to cope with heterogenous investments**

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# Status quo: Energy system models



- **Large-scale optimization models** to model electricity and other energy systems
  - Mostly formulated as linear programs or mixed integer linear programs
- Focus on **generation expansion** and **operations**,
  - Generally limited detail regarding grid modelling
- Examples of long-standing energy system models:
  - **MARKAL** (Fishbone and Abilock, 1981), **TIMES** (Loulou, 2008; Loulou and Labriet, 2008)
  - Traditionally making use of only limited number of time slices (representative hours)
- Other examples
  - E2M2s cf. (Swider and Weber, 2007; Spiecker, Vogel and Weber, 2013)
  - PERSEUS (Rosen, Tietze-Stöckinger and Rentz, 2007)
  - REMIX (Scholz, 2012; Gils et al. 2017)
  - JHSMINE (Munoz et al. 2014, Xu and Hobbs 2021)
  - ➢ Often more detailed temporal resolution, up to hourly resolution
  - ➢ But long computation times and/or limitations regarding computational details

# Extensions to cope with heterogeneity



State of the Art

- **Differentiate technologies and/or applications** within energy system models, e.g.
  - Solar energy use differentiated by orientation of roofs
  - Heating systems by building types
- **Separate detailed modelling of** (geographical) distribution of **renewables and demand**, e.g.
  - Renewable potentials and land use restrictions, e.g. renewable ninja (Pfenninger, Staffel 2016)
  - Models for heating systems in buildings, e.g. HeatSim (Bauermann 2016)
- **Iterative coupling** of **energy demand** and **system models**, e.g.
  - Heating systems & electricity markets (Bauermann et al. 2014)
  - Models of demand flexibility & electricity systems (Misconel et al. 2023)

➢ **Why not integrate them?**

# Consumer decisions as optimization models - discrete choice models

State of the Art

- Category of models popularized by Nobel laureate Daniel McFadden and others
  - Describe optimal choices under stochastic utility
  - Diverse specifications, notably logit and probit models
  - Logit specification presents advantage of analytical formulations
- Standard stochastic utility formulation

$$U_i = V_i + \varepsilon_i$$

  - Consumers choose among alternatives $i$
  - Optimal individual choice: highest sum of observable and stochastic utility
- Corresponding choice probability or in case of binary choices (adoption yes/no)

$$Prob_i = \frac{e^{V_i}}{\sum_j e^{V_j}} \qquad\qquad Prob_i = \frac{1}{1+e^{-V_i}}$$

- Corresponding expected indirect utility function: LogExpSum (cf. Small & Rosen 1981)

$$\mathrm{E}[U_i] = \ln(e^{V_i} + 1)$$

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- Discrete choice models
- Energy system models

} **Choice-based energy system modelling**

**Key advantages**:

- More **„realism"** in energy system models
- Align the „central planner paradigm" of energy system models with the distributed decision making of the energy transition

**Key challenges**:

- **Non-linear** model
- Efficient **solution algorithms**

**Note on terminology:**

- Term coined in analogy to „Choice-based facility location planning" (Müller 2023, based on Benati 1999, Benati and Hansen 2002, Haase 2009, Haase and Müller 2014)

➢ Analogy also in two-level problem structure

# Methodological basis: Agent-based welfare maximization

Mathematical problem formulation

Microeconomic background:

- Equivalence of market outcomes under perfect (or working) competition and optimum reached by social planner
- **Welfare** (in partial equilibrium) corresponds to **sum of consumer**(s) **surplus** and **producer**(s) **surplus**

Basic idea:

1) Formulation of surpluses (respectively money-metric utility) for all agents
2) Summation of surpluses
3) Elimination of transfer payments (and the corresponding prices) in the aggregated surplus

First implementation:

- Focus on transformation and capacity constraints

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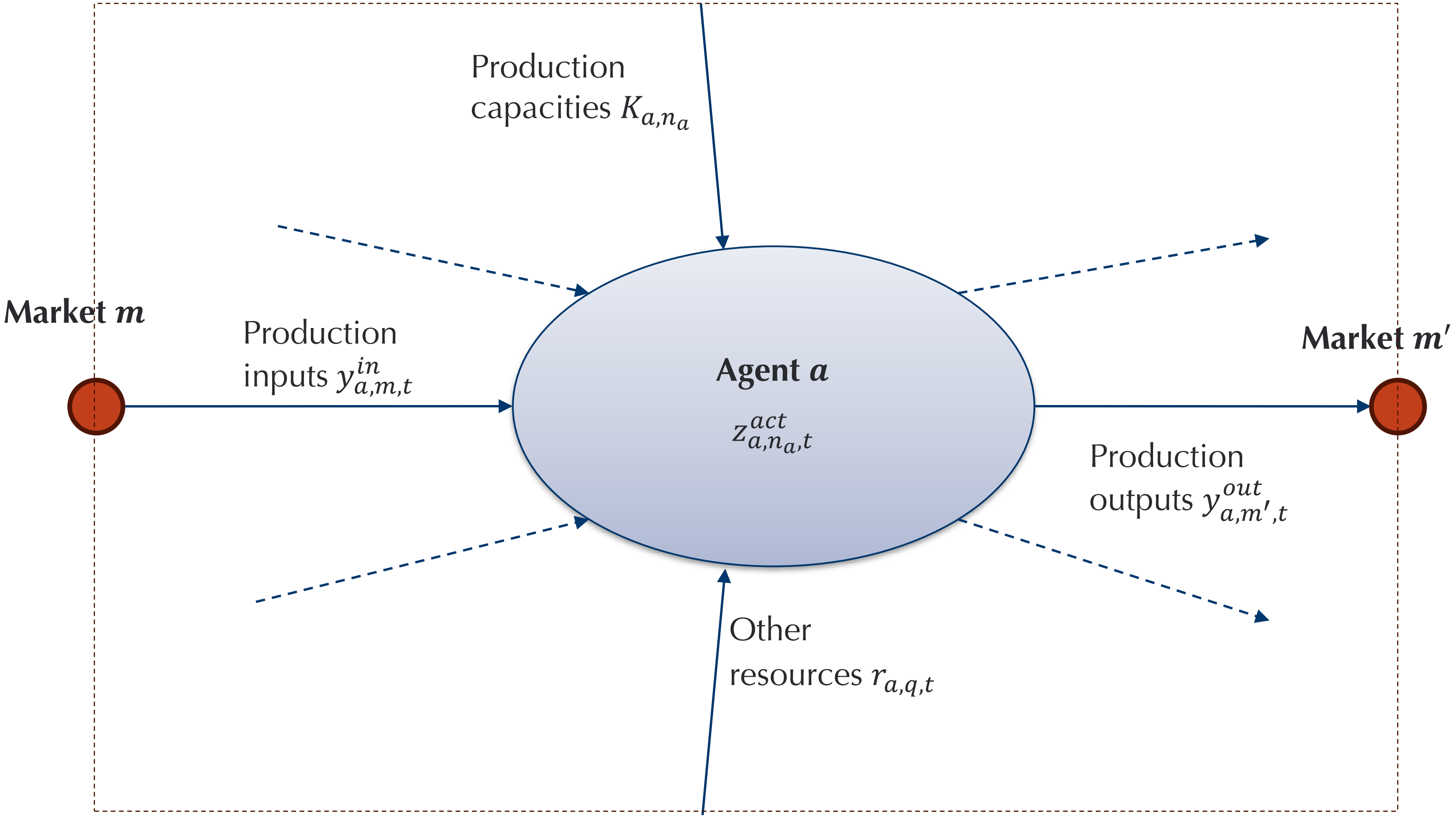
# General agent in energy system models

Mathematical problem formulation




(sub-)system boundary

Source: based on Finke et al. (2024)

# Overall objective function and agent surpluses

Mathematical problem formulation

**Welfare:**

$$\max_{d_t, K_i, y_{i,t}} W$$

$$W = S^C(d_t) + \sum_i S^{CvP}(K_i, y_{i,t}) + \sum_j S_j^{RE}(\pi_j, y_{j,t})$$

**Consumer surplus:**

$$S^C(d_t) = \sum_t (V - p_t) \cdot \Delta t \cdot d_t$$

Utility: represented by VOLL — Cost: based on paid price

**Conventional producer surplus:**

$$S_i^{CvP}(K_i, y_{i,t}) = \sum_t \left(p_t - c_i^{op}\right) \cdot \Delta t \cdot y_{i,t} - c_i^{inv} \cdot K_i$$

Revenue: based on received price — Cost: including operational and investment cost

**Renewable producer surplus:**

$$S_j^{RE}\left(\pi_j, \varphi_{j,t}\right) = \cdots$$

**List of symbols:**

- $i$ index conventional producers
- $j$ index renewable producers
- $t$ index time steps

- $c_i^{op}$ operational cost tech $i$
- $d_t$ (realised) demand time $t$
- $K_i$ capacity tech $i$
- $p_t$ price time $t$
- $S^{xyz}$ economic surplus group xyz
- $V$ value of lost load (VOLL)
- $W$ welfare
- $y_{i,t}$ production tech $i$, time $t$
- $\Delta t$ time step length

# Constraints



## Agent-specific constraints:

Demand:

$$d_t \cdot \Delta t + s_t \cdot \Delta t = D_t \cdot \Delta t \quad \forall t$$

Conventional capacity:

$$y_{i,t} \cdot \Delta t \leq K_i \cdot \Delta t \quad \forall i \, \forall t$$

## Overarching constraints:

Market clearing:

$$d_t \cdot \Delta t = \sum_i y_{i,t} \cdot \Delta t + \sum_j K_j^{max} \varphi_{j,t} \cdot \pi_j \cdot \Delta t - r_t \cdot \Delta t \quad \forall t$$

## List of symbols (continued):

$D_t$ planned demand (load)

$s_t$ load shedding

$K_j^{max}$ maximum potential tech j

$r_t$ renewable curtailment at time t

$\pi_j$ probability of invest in tech j

$\varphi_{j,t}$ generation profile tech j

# Objective function – focus on renewable producer surplus

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## Individual renewable producer profit:

Stochastic term (mean 0)

$$s_j^{RE}(\varepsilon) = \sum_t \left(p_t - c_j^{op}\right) \cdot \Delta t \cdot \varphi_{j,t} - c_j^{inv} - \varepsilon$$

Revenue: based on received price

Cost: including operational and investment cost

## Decision rule individual producer:

$$k_j(\varepsilon) = \mathbf{1}_{s_j^{RE} \geq 0}$$

## Resulting probability of investment:

$$\pi_j = Prob\left(s_j \geq 0\right) = F_\varepsilon\left(\sum_t \left(p_t - c_j^{op}\right) \cdot \Delta t \cdot \varphi_{j,t} - c_j^{inv}\right)$$

## Aggregate renewable producer surplus:

$$S_j^{RE}\left(\pi_j, \varphi_{j,t}\right) = K_j^{max} \int_{-\infty}^{+\infty} k_j(\varepsilon) s_j^{RE}(\varepsilon) f(\varepsilon) d\varepsilon = K_j^{max} \int_{-\infty}^{F_\varepsilon^{-1}(\pi_j)} s_j^{RE}(\varepsilon) f(\varepsilon) d\varepsilon$$

(aggregate) decision variable

$$= \left(\sum_t \left(p_t - c_j^{op}\right) \cdot \Delta t \cdot \varphi_{j,t} - c_j^{inv}\right) K_j^{max} \pi_j - \left(\pi_j \ln \pi_j + \left(1 - \pi_j\right) \ln\left(1 - \pi_j\right)\right) K_j^{max} \frac{1}{\beta}$$

Classical deterministic profitability | Installed capacity | Heterogeneity term (based on entropy function of information theory)

Relevant distribution properties

$$\varepsilon \sim LD(\mu, s)$$

$LD$: logistic distribution

$\mu = 0$: mean

$s = \frac{1}{\beta}$: scale parameter

Cumulative distribution function:

$$F(\varepsilon) = \frac{1}{1 + e^{-\beta\varepsilon}}$$

Probability density function:

$$f(\varepsilon) = \frac{\beta e^{-\beta\varepsilon}}{(1 + e^{-\beta\varepsilon})^2}$$

Note std. deviation:

$$sd = \frac{s\pi}{\sqrt{3}} = \frac{\pi}{\beta\sqrt{3}}$$

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(approach may be generalized to different agent groups)

# Key properties of the aggregate optimization problem

Solution approach

- Proposition 1:

  Aggregation of surpluses of generalised price-taking agents leads to a **non-linear welfare maximization problem** in standard primal variables

- Proposition 2:

  This non-linear welfare maximization problem is **concave**

- Proposition 3:

  The non-linear concave optimization problem may be **reformulated** as an **exponential cone problem**

- Proposition 4 (tentative):

  **Higher heterogeneity**/entropy (as measured by parameter $\frac{1}{\beta}$) increases c.p. welfare if Prob<0.5 and decreases welfare if Prob>0.5

# Proposition 1 – non-linear welfare maximization

**Welfare:**

$$W = S^C(d_t) + \sum_i S^{conv}(K_i, y_{i,t}) + \sum_j S_j^{RE}(\pi_j, y_{j,t})$$

$$= \underbrace{\sum_t (V - p_t) \cdot \Delta t \cdot d_t}_{\text{Consumer surplus}} + \underbrace{\sum_i \left(\sum_t \left(p_t - c_i^{op}\right) \cdot \Delta t \cdot y_{i,t} - c_i^{inv} \cdot K_i\right)}_{\text{Conventional producer surplus}} + \underbrace{\sum_j \left(\sum_t \left(p_t - c_j^{op}\right) \cdot \Delta t \cdot \varphi_{j,t} - c_j^{inv}\right) K_j^{max} \pi_j + \sum_j H(\pi_j) K_j^{max} \frac{1}{\beta}}_{\text{Renewable producer surplus}}$$

Collection of terms with $p_t$:

Def.: $H(\pi_j) = -\left(\pi_j \ln \pi_j + (1 - \pi_j) \ln(1 - \pi_j)\right)$

$$\sum_t \left(p_t \cdot \Delta t \cdot \left(-d_t + \sum_i y_{i,t} + \sum_j \varphi_{j,t} K_j^{max} \pi_j\right)\right) = \sum_t p_t \cdot \Delta t \cdot r_t$$

$= 0$ under standard market and renewable assumptions

**Revised aggregate welfare:**

$$W = -\left(\sum_t \Delta t \cdot \left(V \cdot s_t + \sum_i c_i^{op} y_{i,t}\right) + \sum_i c_i^{inv} K_i + \sum_j c_j^{inv} K_j^{max} \pi_j\right) + \sum_j H(\pi_j) K_j^{max} \frac{1}{\beta}$$

- ➢ elimination of dual variable $p_t$, equivalent to cost minimization corrected for heterogeneity term

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# Proposition 2 – concave problem

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**Objective function:**

$$W = -\left(\sum_t \Delta t \cdot \left(V \cdot s_t + \sum_i c_i^{op} y_{i,t}\right) + \sum_i c_i^{inv} K_i + \sum_j c_j^{inv} K_j^{max} \pi_j\right) + \sum_j H(\pi_j) K_j^{max} \frac{1}{\beta}$$

**Constraints:**

Demand: $$d_t \cdot \Delta t + s_t \cdot \Delta t = D_t \cdot \Delta t \quad \forall t$$

Conventional capacity: $$y_{i,t} \cdot \Delta t \leq K_i \cdot \Delta t \quad \forall i\, \forall t$$

Market clearing: $$d_t \cdot \Delta t = \sum_i y_{i,t} \cdot \Delta t + \sum_j K_j^{max} \varphi_{j,t} \cdot \pi_j \cdot \Delta t - r_t \cdot \Delta t \quad \forall t$$

- Objective function is non-linear in $\pi_j$, everything else is linear

Derivatives:

$$\frac{\partial W}{\partial \pi_j} = -c_j^{inv} K_j^{max} - \left(\ln \pi_j + 1 - \ln(1 - \pi_j) - 1\right) K_j^{max} \frac{1}{\beta}$$

$$\frac{\partial^2 W}{\partial \pi_j^2} = -\left(\frac{1}{\pi_j} + \frac{1}{1 - \pi_j}\right) K_j^{max} \frac{1}{\beta} < 0 \quad \forall \pi_j \in (0,1) \qquad \frac{\partial^2 W}{\partial \pi_j \partial \pi_i} = 0 \ \forall j \neq i$$

- Objective function is concave, as are the constraints

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# Proposition 3 – reformulation with exponential cones

Solution approach

Definition exponential cone:

$$\mathcal{K}_{exp} = \left\{(x_1, x_2, x_3) \middle| x_1 \geq x_2 e^{\frac{x_3}{x_2}}, x_2 > 0\right\} \cup \{(x_1, 0, x_3) | x_1 \geq 0, x_3 \leq 0\}$$

Note equivalence for key inequality:

$$x_1 \geq x_2 e^{\frac{x_3}{x_2}} \Leftrightarrow \ln(x_1) \geq \ln(x_2) + \frac{x_3}{x_2} \Leftrightarrow x_3 \leq x_2 \ln(x_1) - x_2 \ln(x_2)$$

Also

$$\max_{\pi_j} A(\pi_j, \dots) + H(\pi_j) \Leftrightarrow \max_{\pi_j} A(\pi_j, \dots) + E_j + E_j^{cp} \,|E_j + E_j^{cp} \leq H(\pi_j)$$

Then

$$E_j \leq -\pi_j \ln \pi_j \Leftrightarrow (1, \pi_j, E_j) \in \mathcal{K}_{exp}$$

Analoguously

$$E_j^{cp} \leq -\pi_j^{cp} \ln \pi_j^{cp} \Leftrightarrow (1, \pi_j^{cp}, E_j^{cp}) \in \mathcal{K}_{exp}$$

With additional constraint

$$\pi_j + \pi_j^{cp} = 1$$

- **Non-linear term in objective function** may be **replaced by two restrictions on exponential cones**
- A **standard solver**, namely **MOSEK**, may be used to solve the non-linear concave optimization problem

# First application: stylized German model



- Based on data from Poestges et al. (2019), publically available under zenodo: https://zenodo.org/record/3674005
  - Reference year for weather and demand 2015
  - $CO_2$ price 100 €/t $CO_2$
  - Wind energy potentials adjusted to current German law, requiring 2 % of land area to be made available
- Spatial resolution: Germany split in five TSO regions (cf. Figure)
- Temporal resolution: 1 year in 8760 hours
- Investments in the following technologies:
  - CCGT
  - OCGT
  - PV
  - Wind onshore
- No grid restrictions, only small transport fee (0.01 €/MWh)
- Scale parameter for heterogeneity $s = \frac{1}{\beta} = 0.1 \cdot c_j^{inv}$

# Computation statistics



Application

- Computations performed on a Notebook running on Windows (13th Gen Intel(R) Core(TM) i7-1365U, 1.80 GHz, 32 GB RAM)
- Software used: GAMS release 46.2.0 & MOSEK solver version 10.1.27

| Model | Choice-based ESM | Linear ESM |
|---|---|---|
| Variables | 394,291 | 394,231 |
| Constraints | 131,471 | 131,411 |
| Non-zero elements | 1,114,986 | 1,114,866 |
| Exponential cones | 20 | - |
| Iterations | 108 | 33 |
| Optimizer time in s | 20.0 | 6.4 |
| Total time in s | 25.9 | 11.9 |

# Results: Capacities in GW

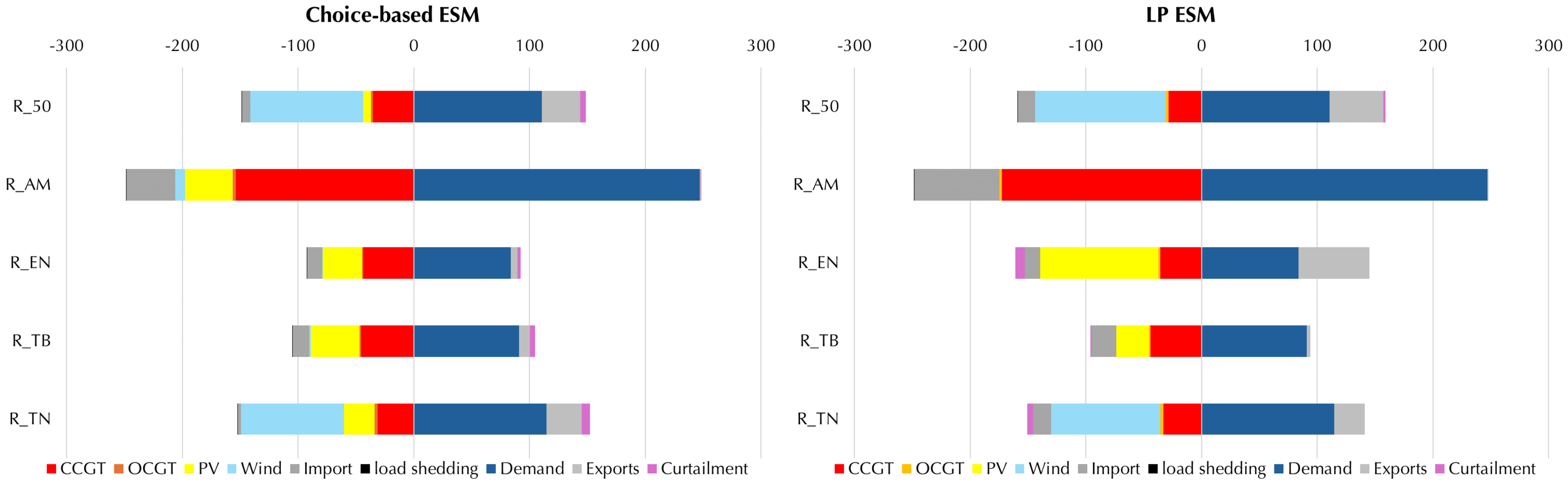


| Capacity PV | | | | | | |
|---|---|---|---|---|---|---|
| | **Total** | **R_50** | **R_AM** | **R_EN** | **R_TB** | **R_TN** |
| **Choice-based ESM** | 137 | 7 | 38 | 29 | 38 | 26 |
| **LP ESM** | 114 | 0 | 0 | 89 | 25 | 0 |
| Max Capacity | 988 | 45 | 291 | 143 | 224 | 285 |
| **Capacity Wind** | | | | | | |
| | **Total** | **R_50** | **R_AM** | **R_EN** | **R_TB** | **R_TN** |
| **Choice-based ESM** | 78 | 38 | 5 | 0 | 1 | 33 |
| **LP ESM** | 80 | 45 | 0 | 0 | 0 | 35 |
| Max Capacity | 138 | 45 | 21 | 12 | 24 | 35 |

| | Capacity gas CCGT Total | Capacity gas OCGT Total |
|---|---|---|
| **Choice-based ESM** | 65 | 25 |
| **LP ESM** | 66 | 25 |

# Regional energy balances

Application



# Cost perspective

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| Cost in b€ | Choice-based ESM | LP ESM | Relative change |
|---|---|---|---|
| investment conventional | 6.51 | 6.53 | -0.3% |
| investment variable RE | 18.70 | 17.56 | 6.5% |
| load shedding | 0.08 | 0.08 | 1.4% |
| operations | 25.45 | 25.89 | -1.7% |
| transport | 0.00 | 0.00 | -42.6% |
| **Total deterministic cost** | **50.74** | **50.06** | **1.4%** |
| value entropy | 2.99 | 0.00 | |
| **Total** | **47.76** | **50.06** | **-4.6%** |

Note:
Total transport quantities in TWh:
CB ESM: 79.5
LP ESM: 138.6

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# Conclusions and next steps



- **Modelling framework** provides an **innovative approach** to model distributed electricity systems
- Copes with heterogeneity based on an established **stochastic utility framework**
- First application **highlights differences** to conventional linear programs
- **Results to be explored further** in more detailed applications
- Heterogeneity **parameters** may be determined from **empirical observations**

# Thank you for your attention!

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# References